

Modern C++ Programming

20. PERFORMANCE OPTIMIZATION I BASIC CONCEPTS

Federico Busato

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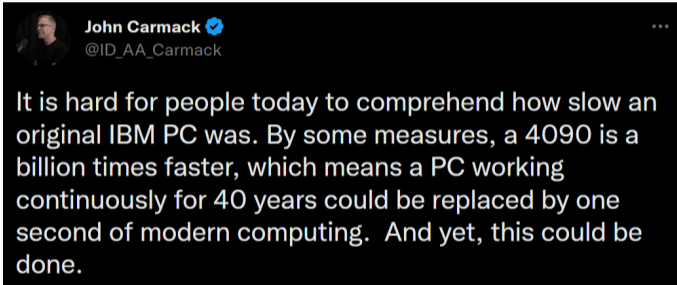
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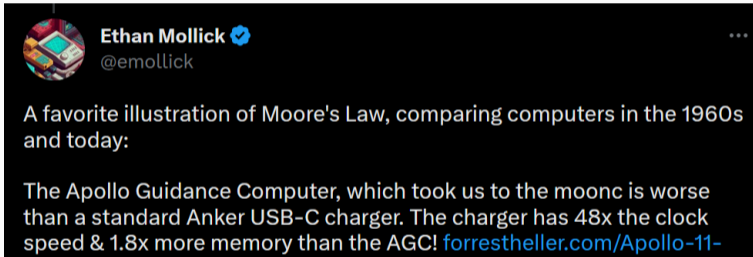
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Introduction

Performance and Technological Progress

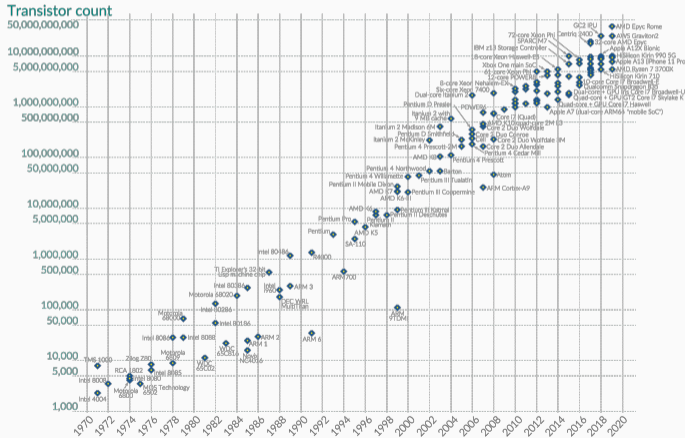


Performance and Technological Progress

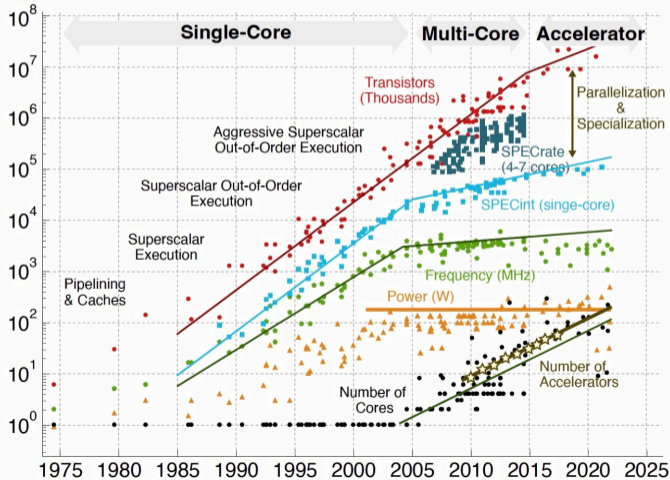


"The number of transistors incorporated in a chip will approximately double every 24 months." (40% per year)

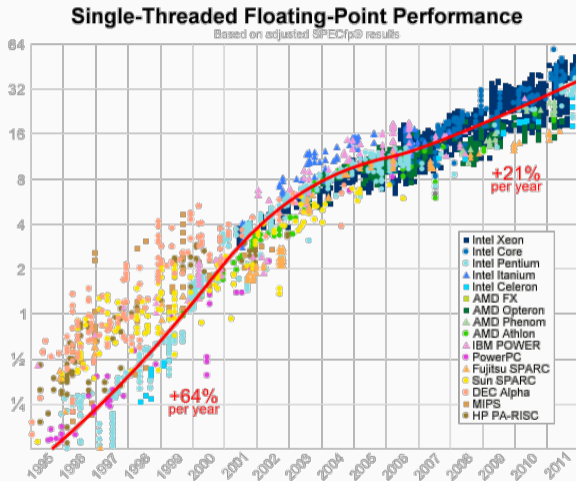
Gordon Moore, Intel co-founder



The Moore's Law is not (yet) dead, but the same concept is not true for *clock frequency, single-thread performance, power consumption, and cost*



Single-Thread Performance Trend





A Look Back at Single-Threaded CPU Performance

Herb Sutter, The Free Lunch Is Over

Higher performance over time is not merely dictated by the number of transistors.

Specific hardware improvements, software engineering, and algorithms play a crucial rule in driving the computer performance.

Technology	<pre>01010011 01100011 01101001 01100101 01101110 01100011 01100101 00000000</pre> 		
	Software	Algorithms	Hardware architecture
Opportunity	Software performance engineering	New algorithms	Hardware streamlining
Examples	Removing software bloat Tailoring software to hardware features	New problem domains New machine models	Processor simplification Domain specialization

Specialized Hardware

Reduced precision, matrix multiplication engine, and sparsity provided orders of magnitude performance improvement for AI applications

Forget Moore's Law. Algorithms drive technology forward

"Algorithmic improvements make more efficient use of existing resources and allow computers to do a task faster, cheaper, or both. Think of how easy the smaller MP3 format made music storage and transfer. That compression was because of an algorithm."

-
- Forget Moore's Law
 - What will drive computer performance after Moore's law?
 - Heeding Huang's Law

Poisson's equation solver on a cube of size $N = n^3$

Year	Method	Reference	Storage	Complexity
1947	GE (banded)	Von Neumann & Goldstine	n^5	$\rightarrow n^7$
1950	Optimal SOR	Reid	n^3	$n^4 \log n$
1971	CG	Young	n^3	$n^{3.5} \log n$
1984	MG	Brandt	n^3	$\rightarrow n^3$

Reasons for Optimizing

- In the first decades, the *computer performance was extremely limited*. Low-level optimizations were essential to fully exploit the hardware
- Modern systems provide much higher performance, but *we cannot more rely on hardware improvement* on short-period
- Performance and efficiency add market value (fast program for a given task), e.g. search, page loading, etc.
- Optimized code uses less resources, e.g. in a program that runs on a server for months or years, a small reduction in the execution time/power consumption translates in a big saving of power consumption

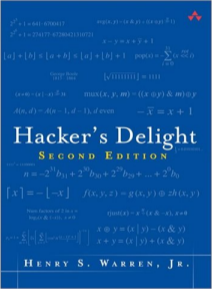
Going the Other Way

- Computing systems are unfathomably complex
- Optimization is complicated and surprising
- Doing something sensible had opposite effect
- We often try clever things that don't work

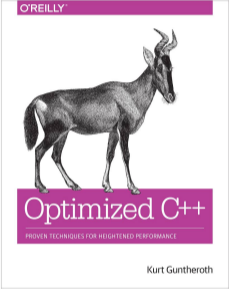
- How about trying something silly then?

25 / 72

from *"Speed is Found in the Minds of People"*,
Andrei Alexandrescu, CppCon 2019



Hacker's Delight (2nd)
H. S. Warren, 2016



Optimized C++
K. Guntheroth, 2014

- `Awesome C/C++ performance optimization resources`, *Bartłomiej Filipek*
- `Optimizing C++`, *wikibook*
- `Optimizing software in C++`, *Agner Fog*
- `Algorithmica: Algorithms for Modern Hardware`
- `What scientists must know about hardware to write fast code`

Basic Concepts

The **asymptotic analysis** refers to estimate the execution time or memory usage as function of the input size (the *order of growing*)

The *asymptotic behavior* is opposed to a *low-level analysis* of the code (instruction/loop counting/weighting, cache accesses, etc.)

Drawbacks:

- The *worst-case* is not the *average-case*
- Asymptotic complexity does not consider small inputs (think to *insertion sort*)
- The hidden constant can be relevant in practice
- Asymptotic complexity does not consider instructions cost and hardware details

Be aware that only **real-world problems** with a small asymptotic complexity or small size can be solved in a “*user*” *acceptable time*

Three examples:

- *Sorting*: $\mathcal{O}(n \log n)$, try to sort an array of some billion elements
- *Diameter of a (sparse) graph*: $\mathcal{O}(V^2)$, just for graphs with a few hundred thousand vertices it becomes impractical without advanced techniques
- *Matrix multiplication*: $\mathcal{O}(N^3)$, even for small sizes N (e.g. 8K, 16K), it requires special accelerators (e.g. GPU, TPU, etc.) for achieving acceptable performance

Time-Memory Trade-off

The **time-memory trade-off** is a way of solving a problem or calculation in less time by using more storage space (less often the opposite direction)

Examples:

- *Memoization* (e.g. used in dynamic programming): returning the cached result when the same inputs occur again
- *Hash table*: number of entries vs. efficiency
- *Lookup tables*: precomputed data instead branches
- *Uncompressed data*: bitmap image vs. jpeg

“If you’re not writing a program, don’t use a programming language”

Leslie Lamport, Turing Award

“First solve the problem, then write the code”

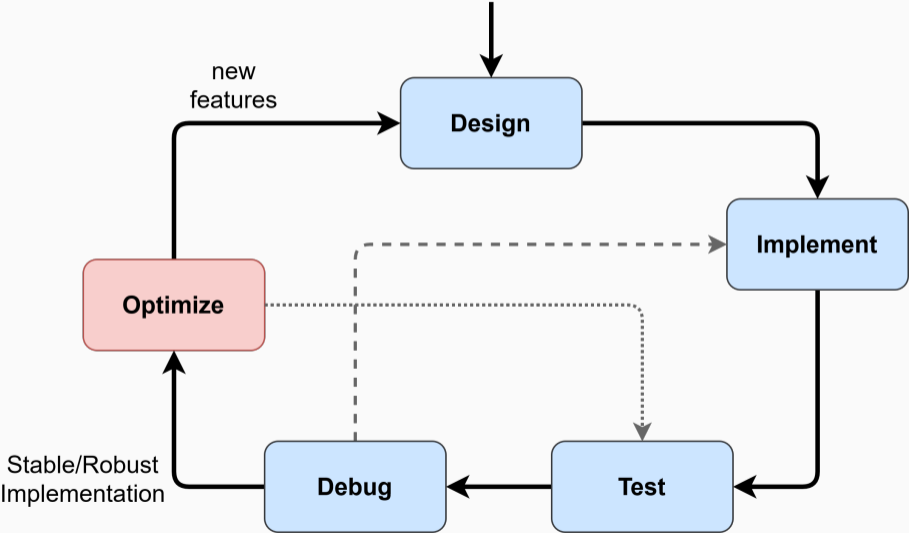
“Inside every large program is an algorithm trying to get out”

Tony Hoare, Turing Award

“Premature optimization is the root of all evil”

Donald Knuth, Turing Award

“Code for correctness first, then optimize!”



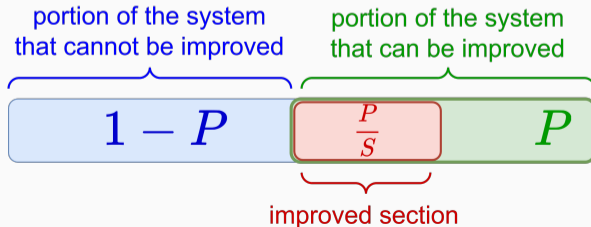
- One of the most important phase of the optimization cycle is the **application profiling** for finding regions of code that are *critical for performance* (**hotspot**)
 - Expensive code region (absolute)
 - Code regions executed many times (cumulative)
- Most of the time, **there is no the perfect algorithm for all cases** (e.g. insertion, merge, radix sort). Optimizing also refers in finding the correct heuristics for different program inputs/platforms instead of modifying the existing code

Ahmdal's Law

The **Ahmdal's law** expresses the maximum improvement possible by improving a particular part of a system

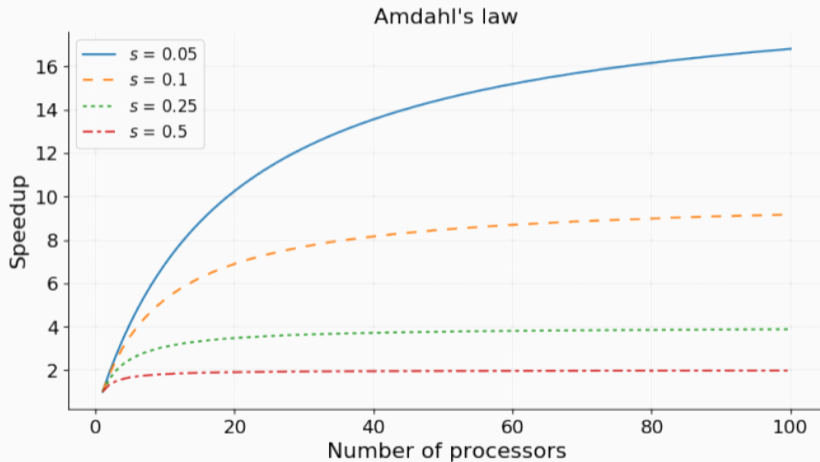
Observation: The performance of any system is constrained by the speed of the slowest point

S : improvement factor expressed as a factor of P



$$\text{Overall Improvement} = \frac{1}{(1 - P) + \frac{P}{S}}$$

P \ S	25%	50%	75%	2x	3x	4x	5x	10x	∞
10%	1.02x	1.03x	1.04x	1.05x	1.07x	1.08x	1.09x	1.10x	1.11x
20%	1.04x	1.07x	1.09x	1.11x	1.15x	1.18x	1.19x	1.22x	1.25x
30%	1.06x	1.11x	1.15x	1.18x	1.25x	1.29x	1.31x	1.37x	1.49x
40%	1.09x	1.15x	1.20x	1.25x	1.36x	1.43x	1.47x	1.56x	1.67x
50%	1.11x	1.20x	1.27x	1.33x	1.50x	1.60x	1.66x	1.82x	2.00x
60%	1.37x	1.25x	1.35x	1.43x	1.67x	1.82x	1.92x	2.17x	2.50x
70%	1.16x	1.30x	1.43x	1.54x	1.88x	2.10x	2.27x	2.70x	3.33x
80%	1.19x	1.36x	1.52x	1.67x	2.14x	2.50x	2.78x	3.57x	5.00x
90%	1.22x	1.43x	1.63x	1.82x	2.50x	3.08x	3.57x	5.26x	10.00x



note: s is the portion of the system that cannot be improved

Throughput, Bandwidth, Latency

The **throughput** is the rate at which operations are performed

Peak throughput:

$(\text{CPU speed in Hz}) \times (\text{CPU instructions per cycle}) \times$
 $(\text{number of CPU cores}) \times (\text{number of CPUs per node})$

NOTE: modern processors have more than one computation unit

The **memory bandwidth** is the amount of data that can be loaded from or stored into a particular memory space

Peak bandwidth:

$(\text{Frequency in Hz}) \times (\text{Bus width in bit} / 8) \times (\text{Pump rate, memory type multiplier})$

The **latency** is the amount of time needed for an operation to complete

The performance of a program is *bounded* by one or more aspects of its computation. This is also strictly related to the underlying hardware

- **Memory-bound.** The program spends its time primarily in performing *memory accesses*. The performance is limited by the *memory bandwidth* (rarely memory-bound also refers to the amount of memory available)
- **Compute-bound** (Math-bound). The program spends its time primarily in computing *arithmetic instructions*. The performance is limited by the *speed of the CPU*

- **Latency-bound.** The program spends its time primarily in waiting *the data are ready* (instruction/memory dependencies). The performance is limited by the *latency of the CPU/memory*
- **I/O Bound.** The program spends its time primarily in performing *I/O operations* (network, user input, storage, etc.). The performance is limited by the *speed of the I/O subsystem*

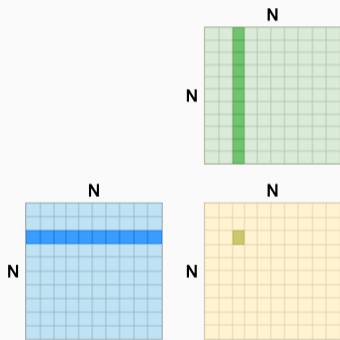
Arithmetic Intensity

Arithmetic/Operational Intensity is the ratio of total operations to total data movement (bytes or words)

The **arithmetic intensity** is a fundamental metric to understand the performance limitations of a system, namely *compute-bound* or *memory-bound*

The **roofline model** uses the *arithmetic intensity* to visually assess the performance of a system and the algorithms/implementations that execute on it

The naive matrix multiplication algorithm requires $N^3 \cdot 2$ floating-point operations* (multiplication + addition) and operates on $(N^2 \cdot 4B) \cdot 3$ data



Considering an ideal system, where each matrix entry is accessed only once, and `float` data type

$$R = \frac{\text{ops}}{\text{bytes}} = \frac{2N^3}{12N^2} = \frac{N}{6}$$

which means that for every byte accessed, the algorithm performs $\frac{N}{6}$ operations \rightarrow **compute-bound**

* What Is a Flop?

Assuming N a large value ($N * N \gg$ cache size), the basic algorithm is equivalent to a dot product for each entry of the output matrix. The algorithm performs $2N^3$ operations and involves $N^3 * 4B$ data movement (excluding storing the results on C)

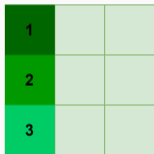
```
for (int i = 0; i < N; i++) {  
    for (int j = 0; j < N; j++) {  
        float sum = 0;  
        for (int k = 0; k < N; k++)  
            sum += A[i][k] * B[k][j]; // row-major order  
        C[i][j] = sum;  
    }  
}
```

$$\frac{\text{ops}}{\text{bytes}} = \frac{2N^3}{12N^3} = \frac{1}{6} \rightarrow \text{memory-bound}$$

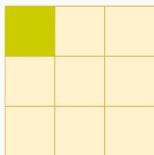
One of the main optimizations in matrix multiplication is to organize the computation by partitioning the matrices into **blocks** (or **tiles**). The primary goal is to take advantage of the memory hierarchy to improve *data locality*

While blocked matrix multiplication doesn't change the number of operations, it *significantly reduces data movement* out of main memory

By selecting blocks of optimal size, we can reduce the data movement by a factor proportional to the block size. The computation can be viewed as a sequence of dot products, one for each block in the output matrix



Considering an optimal block size B to fully exploit the caches



$$\frac{\text{ops}}{\text{bytes}} = \frac{2N^3}{12\left(\frac{N}{B}\right)^3} = \frac{B^3}{6} \rightarrow \text{compute-bound}$$

N	Operations	Data Movement	Ratio	Exec. Time
512	$268 \cdot 10^6$	3 MB	85	2 ms
1024	$2 \cdot 10^9$	12 MB	170	21 ms
2048	$17 \cdot 10^9$	50 MB	341	170 ms
4096	$137 \cdot 10^9$	201 MB	682	1.3 s
8192	$1 \cdot 10^{12}$	806 MB	1365	11 s
16384	$9 \cdot 10^{12}$	3 GB	2730	90 s

A modern CPU performs 100 GFlops, and has about 50 GB/s memory bandwidth

Basic Architecture Concepts

Instruction Throughput, In-Order, and Out-of-Order Execution

The *processor throughput*, namely the number of instructions that can be executed in a unit of time, is measured in **Instruction per Cycle (IPC)**.

It is worth noting that most instructions require multiple clock cycles (**Cycles Per Instruction, CPI**). Therefore improving the IPC requires advanced hardware support

In-Order Execution (IOE) refers to the sequential processing of instructions in the exact order they appear in the program

Out-of-Order Execution (OOE) refers to the execution of instructions based on the availability of input data and execution units, rather than their original order in a program executed in a unit of time

Out-of-order execution on a scalar processor (single instruction at a time) is implemented through **instruction pipelining** which consists in dividing instructions into stages performed by different processor units, allowing different parts of instructions to be processed in parallel

Instruction pipelining breaks up the processing of instructions into several steps, allowing the processor to avoid stalls that occur when the data needed to execute an instruction is not immediately available. The processor avoid stalls by filling slots with other instructions that are ready

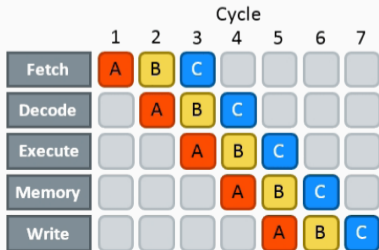
Fetch: The processor retrieves an instruction from memory

Decode: Instruction interpretation and preparation for execution, determining what operations it calls for

Execute: The processor carries out the instruction

Memory Access: Reading from or writing to memory (if needed)

Write-back: The results of the instruction execution are written back to the processor's registers or memory



Microarchitecture	Pipeline stages
Core	14
Bonnell	16
Sandy Bridge	14
Silvermont	14 to 17
Haswell	14
Skylake	14
Kabylake	14

The *pipeline efficiency* is affected by

- **Instruction stalls**, e.g. cache miss, an execution unit not available, etc.
- **Bad speculation**, branch misprediction

A **superscalar processor** is a type of microprocessor architecture that allows for the execution of *multiple instructions in parallel during a single clock cycle*. This is achieved by incorporating multiple execution units within the processor

The concept should not be confused with *instruction pipelining*, which decompose the instruction processing in stages. Modern processors combine both techniques to improve the IPC

Instruction-Level Parallelism (ILP) is a measure of how many instructions in a computer program can be executed simultaneously by issuing *independent* instructions in sequence.

ILP is achieved with *out-of-order execution* or the *SIMT* programming model

```
for (int i = 0; i < N; i++) // with no optimizations, the loop
    C[i] = A[i] * B[i];    // is executed in sequence
```

can be rewritten as:

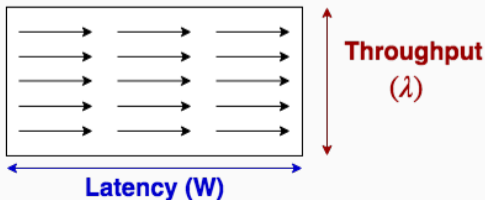
```
for (int i = 0; i < N; i += 4) { // four independent multiplications
    C[i] = A[i] * B[i]; // per iteration
    C[i + 1] = A[i + 1] * B[i + 1]; // A, B, C are not alias
    C[i + 2] = A[i + 2] * B[i + 2];
    C[i + 3] = A[i + 3] * B[i + 3];
}
```

ILP and Little's Law

The **Little's Law** expresses the relation between *latency* and *throughput*. The *throughput* of a system λ is equal to the number of elements in the system divided by the average time spent (*latency*) W for each element in the system:

$$L = \lambda W \quad \rightarrow \quad \lambda = \frac{L}{W}$$

- L : average number of customers in a store
- λ : arrival rate (*throughput*)
- W : average time spent (*latency*)



Data-Level Parallelism (DLP) and SIMD

Data-Level Parallelism (DLP) refers to the execution of the same operation on multiple data in parallel

Vector processors or array processors provide SIMD (*Single Instruction-Multiple Data*) or vector instructions for exploiting data-level parallelism

The popular vector instruction sets are:

MMX *MultiMedia eXtension*. 80-bit width (Intel, AMD)

SSE (SSE2, SSE3, SSE4) *Streaming SIMD Extensions*. 128-bit width (Intel, AMD)

AVX (AVX, AVX2, AVX-512) *Advanced Vector Extensions*. 512-bit width (Intel, AMD)

NEON *Media Processing Engine*. 128-bit width (ARM)

SVE (SVE, SVE2) *Scalable Vector Extension*. 128-2048 bit width (ARM)

Thread-Level Parallelism (TLP)

A **thread** is a single sequential execution flow within a program with its state (instructions, data, PC, register state, and so on)

Thread-level parallelism (TLP) refers to the execution of separate computation “*thread*” on different processing units (e.g. CPU cores)

Single Instruction Multiple Threads (SIMT)

An alternative approach to the classical data-level parallelism is **Single Instruction Multiple Threads (SIMT)**, where multiple threads execute the same instruction simultaneously, with each thread operating on different data.

GPUs are successful examples of SIMT architectures.

SIMT can be thought of as an evolution of *SIMD* (Single Instruction Multiple Data). *SIMD* requires that all data processed by the instruction be of the same type and requires no dependencies or inter-thread communication. On the other hand, **SIMT** is more flexible and does not have these restrictions. Each thread has access to its own memory and can operate independently.

The **Instruction Set Architecture** (ISA) is an abstract model of the CPU to represent its behavior. It consists of addressing modes, instructions, data types, registers, memory architecture, interrupt, etc.

It does not define how an instruction is processed

The **microarchitecture** (μ arch) is the implementation of an **ISA** which includes pipelines, caches, etc.

Complex Instruction Set Computer (CISC)

- Complex instructions for special tasks even if used infrequently
- Assembly instructions follow software. Little compiler effort for translating high-level language into assembly
- Initially designed for saving cost of computer memory and disk storage (1960)
- High number of instructions with different size
- Instructions require complex micro-ops decoding (translation) for exploiting ILP
- Multiple low-level instructions per clock but with high latency

Hardware implications

- High number of transistors
- Extra logic for decoding. Heat dissipation
- Hard to scale

Reduced Instruction Set Computer (RISC)

- Simple instructions
- Small number of instructions with fixed size
- 1 clock per instruction
- Assembly instructions does not follow software
- No instruction decoding

Hardware implications

- High ILP, easy to schedule
- Small number of transistors
- Little power consumption
- Easy to scale

Instruction Set Comparison

x86 Instruction set

```
MOV AX, 15; AH = 00, AL = 0Fh  
AAA; AH = 01, AL = 05  
RET
```

ARM Instruction set

```
MOV R3, # 10  
AND R2, R0, # 0xF  
CMP R2, R3  
IT LT  
BLT elsebranch  
ADD R2, # 6  
ADD R1, #1  
elsebranch:  
END
```

CISC vs. RISC

- **Hardware market:**

- *RISC* (ARM, IBM): Qualcomm Snapdragon, Amazon Graviton, Nvidia Grace, Nintendo Switch, Fujitsu Fukaku, Apple M1, Apple Iphone/Ipod/Mac, Tesla Full Self-Driving Chip, PowerPC
- *CISC* (Intel, AMD): all x86-64 processors

- **Software market:**

- *RISC*: Android, Linux, Apple OS, Windows
- *CISC*: Windows, Linux

- **Power consumption:**

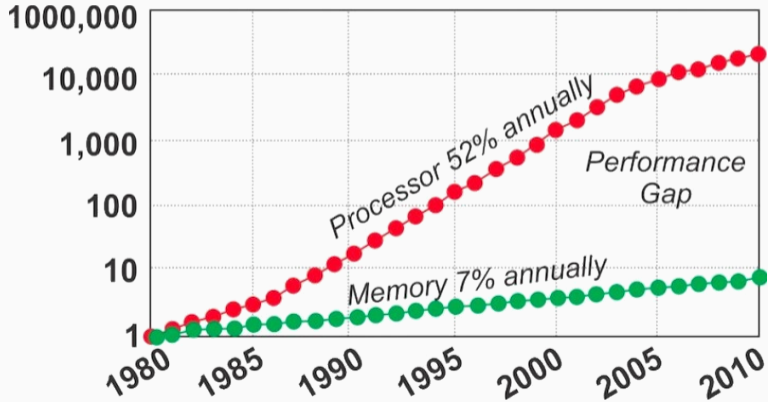
- *CISC*: Intel i5 10th Generation: 64W
- *RISC*: Arm-based smartphone < 5W

“Incidentally, the first ARM1 chips required so little power, when the first one from the factory was plugged into the development system to test it, the microprocessor immediately sprung to life by drawing current from the IO interface – before its own power supply could be properly connected.”

Happy birthday, ARM1. It is 35 years since Britain's Acorn RISC Machine chip sipped power for the first time

Memory Concepts

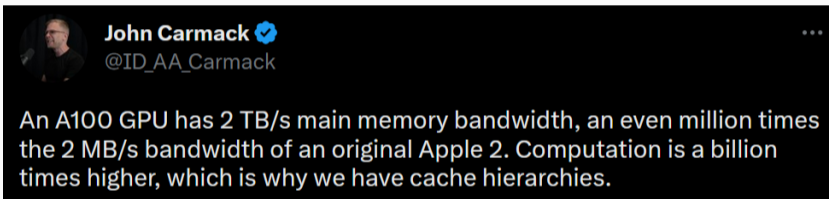
Access to memory dominates other costs in a processor



The Von Neumann Bottleneck

The efficiency of computer architectures is limited by the **Memory Wall** problem, namely the memory is the slowest part of the system

Moving data to and from main memory consumes the vast majority of *time* and *energy* of the system



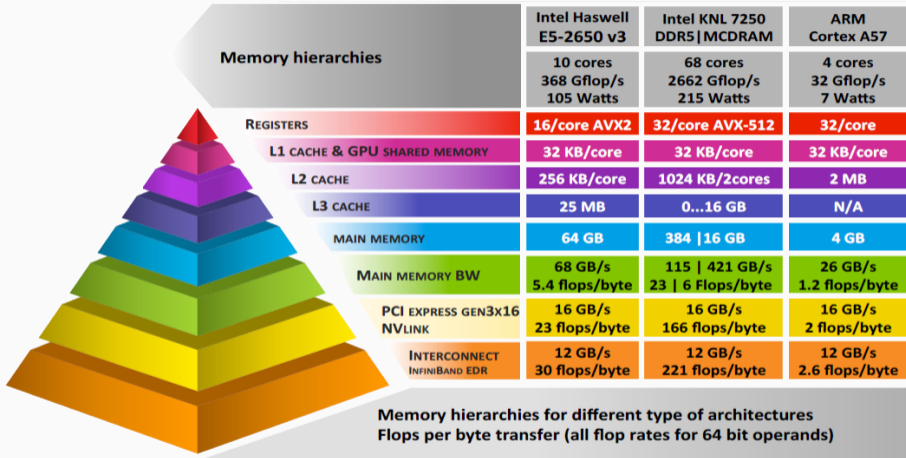
Modern architectures rely on complex memory hierarchy (primary memory, caches, registers, scratchpad memory, etc.). Each level has different characteristics and constrains (size, latency, bandwidth, concurrent accesses, etc.)



1 byte of RAM (1946)



IBM 5MB hard drive (1956)



Source:

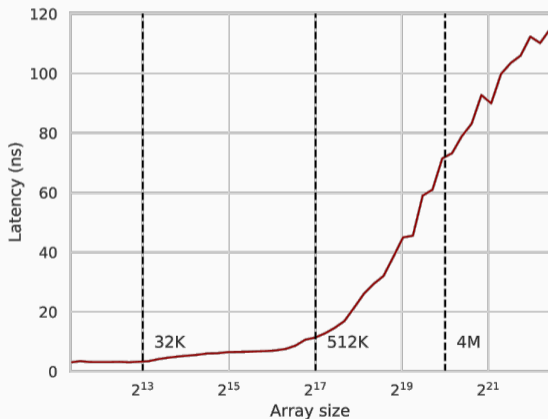
*“Accelerating Linear Algebra on Small Matrices from Batched BLAS to Large Scale Solvers”,
ICL, University of Tennessee*

Intel Alder Lake 12th-gen Core-i9-12900k (Q1'21) + DDR4-3733 example:

Hierarchy level	Size	Latency	Latency Ratio	Bandwidth	Bandwidth Ratio
L1 cache	192 KB	1 ns	1.0x	1,600 GB/s	1.0x
L2 cache	1.5 MB	3 ns	3x	1,200 GB/s	1.3x
L3 cache	12 MB	6 - 20 ns	6-20x	900 GB/s	1.7x
DRAM	/	50 - 90 ns	50-90x	80 GB/s	20x
SDD Disk (swap)	/	70 μ s	10 ⁵ x	2 GB/s	800x
HDD Disk (swap)	/	10 ms	10 ⁷ x	2 GB/s	800x

- en.wikichip.org/wiki/WikiChip
- Memory Bandwidth Napkin Math

“Thinking differently about memory accesses, a good start is to get rid of the idea of $\mathcal{O}(1)$ memory access and replace it with $\mathcal{O}\sqrt{N}$ ” - The Myth of RAM



A **cache** is a small and fast memory located close to the processor that stores frequently used instructions and data. It is part of the processor package and takes 40 to 60 percent of the chip area

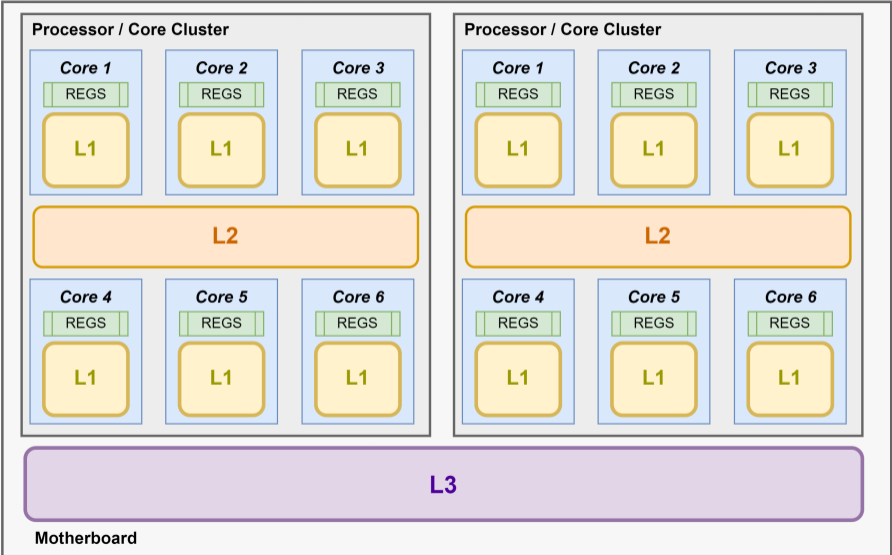
Characteristics and content:

Registers Program counter (PC), General purpose registers, Instruction Register (IR), etc.

L1 Cache Instruction cache and data cache, private/exclusive per CPU core, located on-chip

L2 Cache Private/exclusive per single CPU core or a cluster of cores, located off-chip

L3 Cache Shared between all cores and located off-chip (e.g. motherboard), up to 128/256MB



A **cache line** or **cache block** is the unit of data transfer between the cache and main memory, namely the memory is loaded at the *granularity* of a cache line. A cache line can be further organized in banks or sectors

The typical size of the cache line is 64 bytes on x86-64 architectures (Intel, AMD), while it is 128 bytes on Arm64

Cache access type:

Hot Closest-processor cached, L1

Warm L2 or L3 caches

Cold First load, cache empty

- A **cache hit** occurs when a requested data is *successfully found* in the cache memory
- The **cache hit rate** is the number of *cache hits divided by the number of memory requests*
- A **cache miss** occurs when a requested data is *not found* in the cache memory
- The **miss penalty** refers to the *extra time required to load the data* into cache from the main memory when a cache miss occurs
- A **page fault** occurs when a requested data is in the process address space, but *it is not currently located in the main memory* (swap/pagefile)
- Page **thrashing** occurs when page faults are frequent and the OS spends significant time to swap data in and out the physical RAM

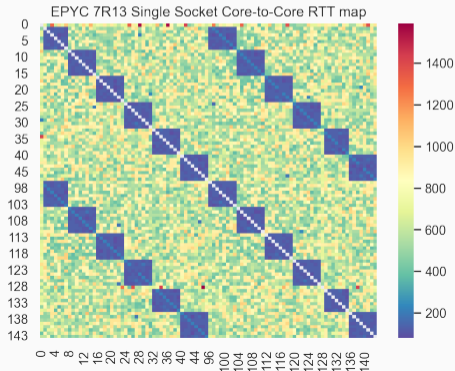
Memory Locality

- **Spatial Locality** refers to the use of data elements within relatively close storage locations e.g. scan arrays in increasing order, matrices by row. It involves mechanisms such as *memory prefetching* and *access granularity*
When spatial locality is low, many words in the cache line are not used
- **Temporal Locality** refers to the reuse of the same data within a relatively small-time duration, and, as consequence, exploit lower levels of the memory hierarchy (caches), e.g. multiple sparse accesses
Heavily used memory locations can be accessed more quickly than less heavily used locations

Core-to-Core Latency

The slowing of Moore's Law and the collapse of Dennard scaling necessitated the hierarchical organization of caches and processors in the CPU. *Today, CPUs organize their cores into clusters, chipllets, and multi-sockets.* As a result, how execution threads are mapped to cores has a significant impact on the overall performance

Core-to-Core
Latency Heatmap:



Thread Affinity

The **thread affinity** refers to the binding of a thread to a specific execution unit. The goal of *thread affinity* is improving the application performance by taking advantage of cache locality and optimizing resource usage

Setting CPU affinity can be done programmatically, such as using the `pthread_setaffinity_np` function for POSIX threads, or at OS level with the `taskset` command and the `sched_setaffinity` system call on Linux

***Dennard Scaling**: power is proportional to the area of the transistor

CPU Affinity: Because Even A Single Chip Is Nonuniform

Memory Ordering Model

- **Source code order:** The order in which the memory operations are specified in the source code, e.g. *subscript, dereferencing*
- **Program order:** The order in which the memory operations are specified at assembly level. Compilers can reorder instructions as part of the optimization process
- **Execution order:** The order in which the individual memory-reference instructions are executed on a given CPU, e.g., *out-of-order execution*
- **Perceived order:** The order in which a CPU perceives its memory operations. The perceived order can differ from the execution order due to caching, interconnect, and memory-system optimizations