Modern C++ Programming

2. Basic Concepts I

- Fundamental Types

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Preparation

What Compiler Should I Use?

Most popular compilers:

- Microsoft Visual Code (**MSVC**) is the compiler offered by Microsoft
- The GNU Compiler Collection (**GCC**) contains the most popular C++ Linux compiler
- **Clang** is a C++ compiler based on LLVM Infrastructure available for Linux/Windows/Apple (default) platforms

Suggested compiler for beginner: **Clang**

- Comparable performance with GCC/MSVC and low memory usage
- Expressive diagnostics (examples and propose corrections)
- Strict C++ compliance. GCC/MSVC compatibility (inverse direction is not ensured)
- Includes very useful tools: memory sanitizer, static code analyzer, automatic formatting, linter, etc.

Install the Compiler on Linux

Install the last gcc/g++ (v11)

```
$ sudo add-apt-repository ppa:ubuntu-toolchain-r/test
$ sudo apt update
$ sudo apt install gcc-11 g++-11
$ gcc-11 --version
```

Install the last clang/clang++ (v13)

```
$ wget https://github.com/llvm/llvm-project/releases/download/\
llvmorg-13.0.0/clang+llvm-13.0.0-x86_64-linux-gnu-ubuntu-20.04.tar.xz
$ tar xf clang+llvm-13.0.0-x86_64-linux-gnu-ubuntu-20.04.tar.xz
$ PATH=$PATH:$(pwd)/bin
$ LD_LIBRARY_PATH=$LD_LIBRARY_PATH:$(pwd)/lib64
$ clang-13 --version
```

Install the Compiler on Windows

Microsoft Visual Studio

- Direct Installer: `Visual Studio Community 2019`

Clang on Windows

Two ways:

- Windows Subsystem for Linux (WSL)
  - `Run` → `optionalfeatures`
  - Select `Windows Subsystem for Linux`, `Hyper-V`, `Virtual Machine Platform`
  - `Run` → `ms-windows-store:` → Search and install `Ubuntu 18.04 LTS`
- Clang + MSVC Build Tools
  - Download Build Tools per Visual Studio
  - Install `Desktop development with C++`

What Editor/IDE Compiler Should I Use?

Popular C++ IDE (Integrated Development Environment):

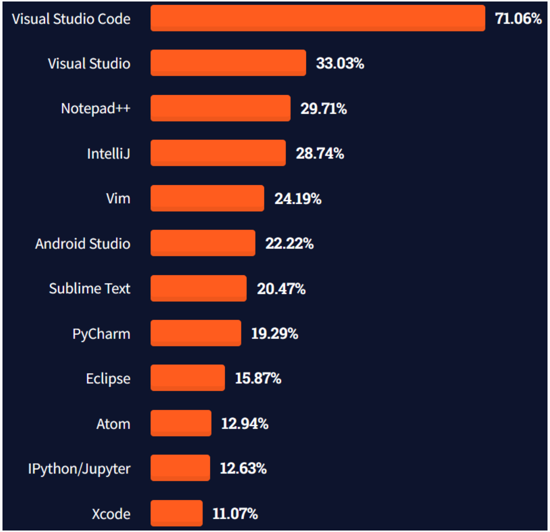
- **Microsoft Visual Studio** (MSVC) (link). Most popular IDE for Windows
- **Clion** (link). (free for student). Powerful IDE with a lot of options
- **QT-Creator** (link). Fast (written in C++), simple
- **XCode**. Default on Mac OS
- **Cevelop** (Eclipse) (link)

Standalone editors for coding (multi-platform):

- **Microsoft Visual Studio Code** (VSCode) (link)
- **Atom** (link) by GitHub/Microsoft
- **Sublime Text editor** (link), written in C++
- **Vim**. Powerful, but needs expertise

Not suggested: `Notepad, Gedit, and other similar editors` (lack of support for programming)

What Editor/IDE Compiler Should I Use?



StackOverflow Developer Survey 2021

How to Compile?

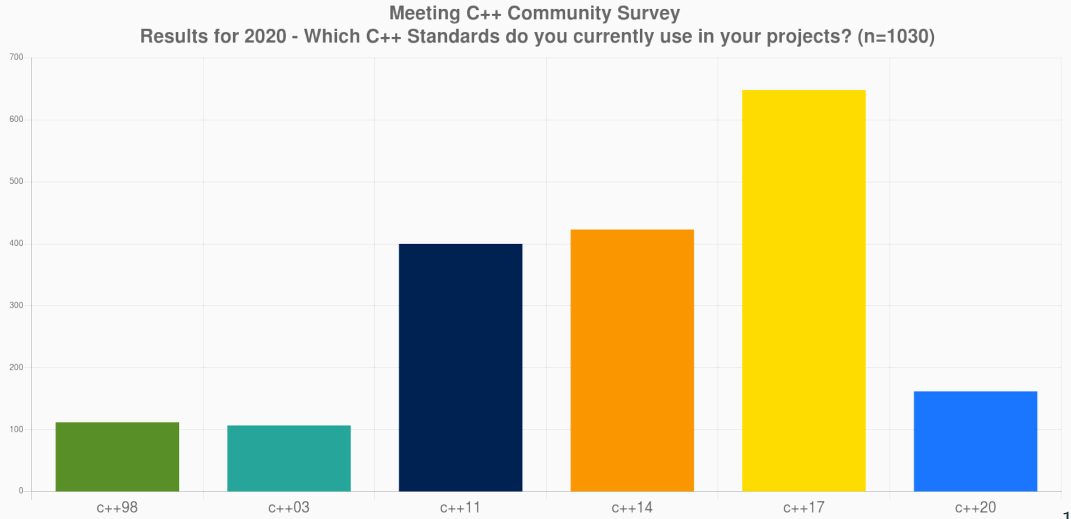
Compile C++11, C++14, C++17, C++20 programs:

```
g++ -std=c++11 <program.cpp> -o program
g++ -std=c++14 <program.cpp> -o program
g++ -std=c++17 <program.cpp> -o program
g++ -std=c++20 <program.cpp> -o program
```

Compiler	C++11		C++14		C++17		C++20	
	Core	Library	Core	Library	Core	Library	Core	Library
g++	4.8.1	5.1	5.1	5.1	7.1	9.0	11+	11+
clang++	3.3	3.3	3.4	3.5	5.0	11.0	12+	14+
MSVC	19.0	19.0	19.10	19.0	19.15	19.15	19.29+	19.29

en.cppreference.com/w/cpp/compiler_support

C++ Standard


Meeting C++ Community Survey
Results for 2020 - Which C++ Standards do you currently use in your projects? (n=1030)
700
600
500
400
300
200
100
0
c++98
c++03
c++11
c++14
c++17
c++20

Hello World

C code with `printf`:

```
#include <stdio.h>

int main() {
    printf("Hello World!\n");
}
```

`printf`
prints on standard output

C++ code with `streams`:

```
#include <iostream>

int main() {
    std::cout << "Hello World!\n";
}
```

`cout`
represent the standard output stream

Hello World

The previous example can be written with the global `std` namespace:

```
#include <iostream>
using namespace std;

int main() {
    cout << "Hello World!\n";
}
```

Note: For sake of space and for improving the readability, we intentionally omit the `std` namespace in the next slides `std::cout` is an example of *output* stream. Data is redirected to a destination, in this case the destination is the standard output

C:

```c
#include <stdio.h>
int main() {
    int    a   = 4;
    double b   = 3.0;
    char   c[] = "hello";
    printf("%d %f %s\n", a, b, c);
}
```

C++:

```cpp
#include <iostream>
int main() {
    int    a   = 4;
    double b   = 3.0;
    char   c[] = "hello";
    std::cout << a << " " << b << " " << c << "\n";
}
```

I/O Stream (Why should we prefer I/O stream?) 2/3

- **Type-safe**: The type of object pass to I/O stream is known statically by the compiler. In contrast, `printf` uses `%` fields to figure out the types dynamically
- **Less error prone**: With IO Stream, there are no redundant `%` tokens that have to be consistent with the actual objects pass to I/O stream. Removing redundancy removes a class of errors
- **Extensible**: The C++ IO Stream mechanism allows new user-defined types to be pass to I/O stream without breaking existing code
- **Comparable performance**: If used correctly may be faster than C I/O (`printf`, `scanf`, etc.) .

I/O Stream (Common C errors) 3/3

- Forget the number of parameters:

```
printf("long phrase %d long phrase %d", 3);
```

- Use the wrong format:

```
int a = 3;
...many lines of code...
printf(" %f", a);
```

- The `%c` conversion specifier does not automatically skip any leading white space:

```
scanf("%d", &var1);
scanf(" %c", &var2);
```

Fundamental Types Overview

Arithmetic Types

Type	Bytes	Range	Fixed width types
`bool`	1	true, false	
`char` †	1	-127 to 127	
`signed char`	1	-128 to 127	int8_t
`unsigned char`	1	0 to 255	uint8_t
`short`	2	-2^{15} to $2^{15}-1$	int16_t
`unsigned short`	2	0 to $2^{16}-1$	uint16_t
`int`	4	-2^{31} to $2^{31}-1$	int32_t
`unsigned int`	4	0 to $2^{32}-1$	uint32_t
`long int`	4/8*		int32_t/int64_t
`long unsigned int`	4/8*		uint32_t/uint64_t
`long long int`	8	-2^{63} to $2^{63}-1$	int64_t
`long long unsigned int`	8	0 to $2^{64}-1$	uint64_t
`float` (IEEE 754)	4	$\pm 1.18 \times 10^{-38}$ to $\pm 3.4 \times 10^{+38}$	
`double` (IEEE 754)	8	$\pm 2.23 \times 10^{-308}$ to $\pm 1.8 \times 10^{+308}$	

* 4 bytes on Windows64 systems, † one-complement

Arithmetic Types - Short Name

Signed Type	short name
`signed char`	/
`signed short int`	`short`
`signed int`	`int`
`signed long int`	`long`
`signed long long int`	`long long`

Unsigned Type	short name
`unsigned char`	/
`unsigned short int`	`unsigned short`
`unsigned int`	`unsigned`
`unsigned long int`	`unsigned long`
`unsigned long long int`	`unsigned long long`

Arithmetic Types - Suffix and Prefix

Type	SUFFIX	example
`int`	/	2
`unsigned int`	u	3u
`long int`	l	8l
`long unsigned`	ul	2ul
`long long int`	ll	4ll
`long long unsigned int`	ull	7ull
`float`	f	3.0f
`double`		3.0

Representation	PREFIX	example
`Binary` C++14	0b	0b010101
`Octal`	0	0308
`Hexadecimal`	0x or 0X	0xFFA010

C++14 allows also *digit separators* for improving the readability `1'000'000`

Other Arithmetic Types

- C++ provides also `long double` (no IEEE-754) of size 8/12/16 bytes depending on the implementation

- C++ (until C++23*) does not provide support for **16-bit float** data type (IEEE 754-2008)
  - Some compilers already provide support for half float (GCC for ARM: `__fp16`, LLVM compiler: `half`)
  - Some modern CPUs (+ Nvidia GPUs) provide half-float instructions
  - Software support (OpenGL, Photoshop, Lightroom, half.sourceforge.net)

- C++ does not provide support for **128-bit integers** even if some architectures support it. clang and gcc allow 128-bit integers as compiler extension (`__int128`)

* Extended floating-point types and standard names

`size_t` and `std::byte`

`size_t` <cstddef>

`size_t` is an *alias* data type capable of storing the biggest representable value on the current architecture

- `size_t` is an unsigned integer type (of at least 16-bit)
- In common C++ implementations:
  - `size_t` is 4 bytes on 32-bit architectures
  - `size_t` is 8 bytes on 64-bit architectures
- `size_t` is commonly used to represent size measures

C++17 defines also **`std::byte`** type to represent a collection of bit (`<cstddef>`). It supports only bitwise operations (no conversions or arithmetic operations)

`void` Type

`void` is an incomplete type (not defined) without a value

- `void` indicates also a function has no return type
  e.g. `void f()`

- `void` indicates also a function has no parameters
  e.g. `f(void)`

- In C `sizeof(void) == 1` (GCC), while in C++ `sizeof(void)` does not compile!!

```
int main() {
//  sizeof(void); // compile error
}
```

Pointer Type

The **type of a pointer** (e.g. `void*`) is an *unsigned* integer of 32-bit/64-bit depending on the underlying architecture

- It only supports the operators `+, -, ++, --` and comparisons `==, !=, <, <=, >, >=`

- A pointer can be *explicitly* converted to an integer type

```
void*  x;
size_t y = (size_t) x; // ok (explicit)
// size_t y = x;        // compile error (implicit)
```

nullptr Keyword

C++11 introduces the new keyword `nullptr` to represent `null` pointers (instead of the `NULL` macro)

```
int* p1 = NULL;       // ok, equal to int* p1 = 0l
int* p2 = nullptr;    // ok, nullptr is a pointer not a number

int n1 = NULL;        // ok, we are assigning 0 to n1
// int n2 = nullptr; // compile error we are assigning
//                      a null pointer to an integer variable

// int* p2 = true ? 0 : nullptr; // compile error
                                 // incompatible types
```

Remember: `nullptr` is not a pointer, but an object of type `nullptr_t` → safer

Fundamental Types Summary

The *fundamental types*, also called *primitive* or *built-in*, are organized into three main categories:

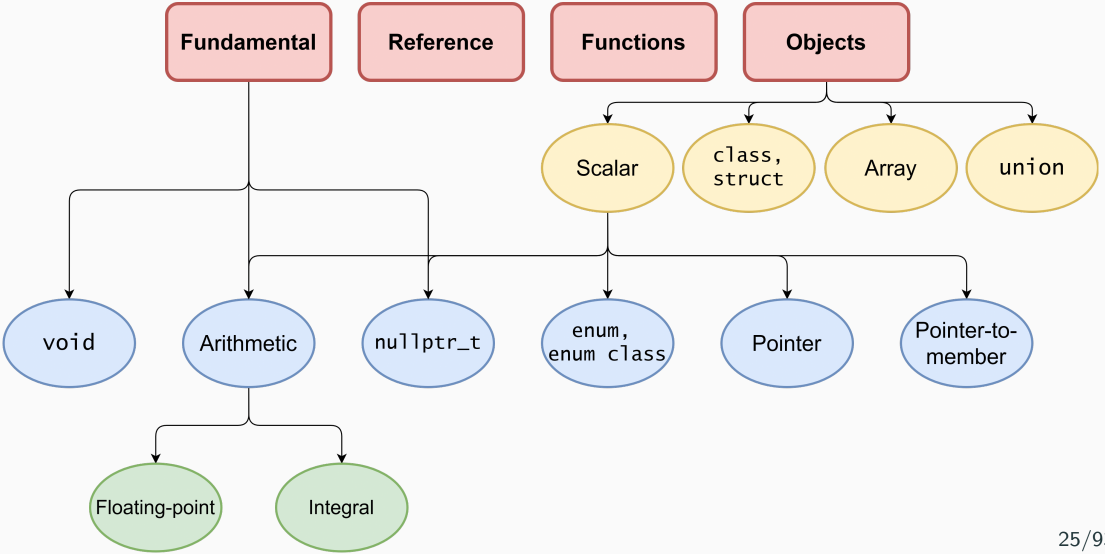
- Integer
- Floating-point
- `void`

Any other entity in C++ is

- an *alias* to the correct type depending to the context and the architectures
- a *composition* of builtin types: `struct/class`, `array`, `union`

`en.cppreference.com/w/cpp/language/types`
`en.cppreference.com/w/cpp/types/integer`

C++ Types Summary



Conversion Rules

Conversion Rules

Implicit type conversion rules (applied in order) :

$\otimes$: any operation (*, +, /, -, %, etc.)

(A) Floating point promotion

`floating_type` $\otimes$ `integer_type` $\rightarrow$ `floating_type`

(B) Implicit integer promotion

`small_integral_type` := any signed/unsigned integral type smaller than `int`

`small_integral_type` $\otimes$ `small_integral_type` $\rightarrow$ `int`

(C) Size promotion

`small_type` $\otimes$ `large_type` $\rightarrow$ `large_type`

(D) Sign promotion

`signed_type` $\otimes$ `unsigned_type` $\rightarrow$ `unsigned_type`

Examples and Common Errors

```
float    f = 1.0f;
unsigned u = 2;
int      i = 3;
short    s = 4;
uint8_t  c = 5; // unsigned char

f * u; // float × unsigned → float: 2.0f
s * c; // short × unsigned char → int: 20
u * i; // unsigned × int → unsigned: 6u
+c;    // unsigned char → int: 5
```

Integers are not floating points!

```
int   b = 7;
float a = b / 2;   // a = 3 not 3.5!!
int   a = b / 2.0; // again a = 3 not 3.5!!
```

C++ Operators

Operators Precedence

Precedence	Operator	Description	Associativity
1	a++ a--	Suffix/postfix increment and decrement	Left-to-right
2	++a --a ! ∼	Prefix increment/decrement, Logical/Bitwise Not	Right-to-left
3	a*b a/b a%b	Multiplication, division, and remainder	Left-to-right
4	a+b a-b	Addition and subtraction	Left-to-right
5	≪ ≫	Bitwise left shift and right shift	Left-to-right
6	< <= > >=	Relational operators	Left-to-right
7	== !=	Equality operators	Left-to-right
8	&	Bitwise AND	Left-to-right
9	^	Bitwise XOR	Left-to-right
10	|	Bitwise OR	Left-to-right
11	&&	Logical AND	Left-to-right
12	||	Logical OR	Left-to-right

- **Unary** operators have higher precedence than **binary operators**
- **Standard math operators** (`+, *, etc.`) have higher precedence than **comparison**, **bitwise**, and **logic** operators
- **Comparison** operators have higher precedence than **bitwise** and **logic operators**
- **Bitwise** operators have higher precedence than **logic** operators
- **Compound assignment** operators `+=`, `-=`, `*=`, `/=`, `%=`, `^=`, `!=`, `&=`, `>>=`, `<<=` have lower priority
- The **comma** operator has the lowest precedence (see next slides)

en.cppreference.com/w/cpp/language/operator_precedence

Operators Precedence

Examples:

```
a + b * 4;              // a + (b * 4)

a * b / c % d;          // ((a * b) / c) % d

a + b < 3 >> 4;         // (a + b) < (3 >> 4)

a && b && c || d;       // (a && b && c) || d

a | b & c || e && d;    // ((a | (b & c)) || (e && d)
```

Important: sometimes parenthesis can make expression worldly... but they can help!

Prefix/Postfix Increment Semantic

Prefix Increment/Decrement `++i`, `--i`

(1) Update the value

(2) Return the new (updated) value

Postfix Increment/Decrement `i++`, `i--`

(1) Save the old value

(2) Update the value

(3) Return the old (original) value

Prefix/Postfix increment/decrement semantic applies not only to built-in types but also to objects

Operation Ordering Undefined Behavior

Expressions with undefined (implementation-defined) behavior:

```
int i = 0;
i = ++i + 2;        // until C++11: undefined behavior
                    // since C++11:  i = 3
i = 0;
i = i++ + 2;        // until C++17: undefined behavior
                    // since C++17: i = 3

f(i = 2, i = 1);    // until C++17: undefined behavior
                    // since C++17:  i = 2
i = 0;
a[i] = i++;         // until C++17: undefined behavior
                    // since C++17: a[1] = 1

f(++i, ++i);        // undefined behavior
i = ++i + i++;      // undefined behavior
```

Assignment, Compound, and Comma Operators

Assignment and **compound assignment** operators have *right-to-left associativity* and their expressions return the assigned value

```
int y = 2;
int x = y = 3; // y=3, then x=3
               // the same of x = (y = 3)
if (x = 4)     // assign x=4 and evaluate to true
```

The **comma** operator has *left-to-right associativity*. It evaluates the left expression, discards its result, and returns the right expression

```
int x = (3, 4); // discards 3, then x=4
int y = 0;
int z;
z = y, x;       // z=y (0), then returns x (4)
```

Structure Binding

Structure Binding declaration C++17 binds the specified names to elements of initializer:

```
struct A {
    int x = 1;
    int y = 2;
} a;

auto [x1, y1] = a; // x1=1, y1=2

int b[2] = {1,2};
auto [x2, y2] = b; // x2=1, y2=2

A f() { return A{4, 5}; }
auto [x3, y3] = f(); // x3=4, y4=5
```

Spaceship Operator <=>

C++20 provides the **three-way comparison operator** `<=>`, also called *spaceship operator*, which allows comparing two objects in a similar way of `strcmp`. The operator returns an object that can be directly compared with a positive, 0, or negative integer value

```
(3 <=> 5)     == 0; // false
('a' <=> 'a') == 0; // true

(3 <=> 5)      < 0;  // true
(7 <=> 5)      < 0;  // false
```

The semantic of the *spaceship operator* can be extended to any object (see next lectures) and can greatly simplify the comparison operators overloading

Integral Data Types

A Firmware Bug

> *"Certain SSDs have a firmware bug causing them to irrecoverably fail after exactly 32,768 hours of operation. SSDs that were put into service at the same time will fail simultaneously, so RAID won't help"*

`HPE SAS Solid State Drives - Critical Firmware Upgrade`

Via `twitter.com/martinkl/status/1202235877520482306?s=19`

Overflow Implementations

Google AI Blog

The latest news from Google AI

Extra, Extra - Read All About It: Nearly All Binary Searches and Mergesorts are Broken

Friday, June 2, 2006

Posted by Joshua Bloch, Software Engineer

other examples: average, ceiling division, rounding division

`ai.googleblog.com/2006/06/extra-extra-read-all-about-it-nearly.html`

Potentially Catastrophic Failure

$$51\ days = 51 \cdot 24 \cdot 60 \cdot 60 \cdot 1000 = 4\,406\,400\,000\ ms$$

Boeing 787s must be turned off and on every 51 days to prevent 'misleading data' being shown to pilots

C++ Data Model

LP32 Windows 16-bit APIs (no more used)
ILP32 Windows 32-bit APIs, Unix 32-bit (Linux, Mac OS)
LLP64 Windows 64-bit APIs
LP64 Linux 64-bit APIs

Model/Bits	`short`	`int`	`long`	`long long`	`pointer`
ILP32	16	32	32	64	32
LLP64	16	32	32	64	64
LP64	16	32	64	64	64

`char` is always 1 byte

int*_t <cstdint>

C++ provides fixed width integer types.

They have the same size on any architecture:

int8_t, uint8_t

int16_t, uint16_t

int32_t, uint32_t

int64_t, uint64_t

Good practice: Prefer fixed-width integers instead of native types. `int` and `unsigned` can be directly used as they are widely accepted by C++ data models

Fixed Width Integers 2/3

`int*_t` types are not "real" types, they are merely *typedefs* to appropriate fundamental types

C++ standard does not ensure an one-to-one mapping:

- There are **five** distinct *fundamental types* (`char`, `short`, `int`, `long`, `long long`)
- There are **four** `int*_t` *overloads* (`int8_t`, `int16_t`, `int32_t`, and `int64_t`)

ithare.com/c-on-using-int_t-as-overload-and-template-parameters

Warning: I/O Stream interprets `uint8_t` and `int8_t` as `char` and not as integer values

```
int8_t var;
cin >> var;  // read '2'
cout << var; // print '2'
int a = var * 2;
cout << a;   // print '100' !!
```

When Use Signed/Unsigned Integer? 1/3

`Signed` and `unsigned` integers use the same hardware for their operations, but they have very different semantic:

`signed` integers

- Represent positive, negative, and zero values ($\mathbb{Z}$)
- More negative values ($2^{31} - 1$) than positive ($2^{31} - 2$)
- Overflow/underflow is undefined behavior
  Possible behavior:
  overflow: $(2^{31} - 1) + 1 \rightarrow min$
  underflow: $-2^{31} - 1 \rightarrow max$
- Bit-wise operations are implementation-defined
- Commutative, reflexive, not associative (overflow)

`unsigned` integers

- Represent only *non-negative* values ($\mathbb{N}$)
- Overflow/underflow is well-defined (modulo 2^{32})
- Discontinuity in 0, $2^{32} - 1$
- Bit-wise operations are well-defined
- Commutative, reflexive, associative

Google Style Guide

> Because of historical accident, the C++ standard also uses unsigned integers to represent the size of containers - many members of the standards body believe this to be a mistake, but it is effectively impossible to fix at this point

Solution: use `int64_t`

max value: $2^{63} - 1 =$ 9,223,372,036,854,775,807 or
9 quintillion (9 billion of billion),
about 292 years (nanoseconds),
9 million terabytes

`Subscripts and sizes should be signed`, WG21 P1428R0, *Bjarne Stroustrup*

Arithmetic Type Limits

Query properties of arithmetic types in C++11:

```
#include <limits>

std::numeric_limits<int>::max();      // 2^31 - 1
std::numeric_limits<uint16_t>::max(); // 65,535

std::numeric_limits<int>::min();      // -2^31
std::numeric_limits<unsigned>::min(); // 0
```

* this syntax will be explained in the next lectures

Promotion and Truncation

Promotion to a larger type keeps the sign

```
int16_t x = -1;
int     y = x; // sign extend
cout << y;     // print -1

int64_t    z  = 4294967296; // 2^32 ok
// int64_t z1 = 1 << 32;    // wrong!! z is (potentially) 0
                            // (1) signed shift, (2) shift > bits
```

Truncation to a smaller type is implemented as a modulo operation with respect to the number of bits of the smaller type

```
int     x = 65537; // 2^16 + 1
int16_t y = x;     // x % 2^16
cout << y;         // print 1

int     z = 32769; // 2^15 + 1
int16_t w = z;     // (int16_t) (x % 2^16)
cout << w;         // print -32767
```

Implicit Promotion

Integral data types smaller than 32-bit are *implicitly* promoted to `int`, independently if they are *signed* or *unsigned*

- Unary `+, -, ~` and Binary `+, -, &, etc.` promotion:

```
char a = 48;       // '0'
cout << a;         // print '0'
cout << +a;        // print '48'
cout << (a + 0); // print '48'

uint8_t a1 = 255;
uint8_t b1 = 255;
cout << (a1 + b1);  // print '510' (no overflow)
```

```
unsigned a = 10;  // array is small
int      b = -1;
array[10ull + a * b] = 0; // ?
```

☠ Segmentation fault!

```
int f(int a, unsigned b, int* array) {  // array is small
    if (a > b)
        return array[a - b]; // ?
    return 0;
}
```

☠ Segmentation fault!

```
// v.size() return unsigned
for (size_t i = 0; i < v.size() - 1; i++)
    array[i] = 3; // ?
```

☠ Segmentation fault for v.size() = 0!

Mixing Signed/Unsigned Errors

Easy case:

```
unsigned x = 32;      // x can be also a pointer
x          += 2u - 4; // 2u - 4 = 2 + (2^32 - 4)
                      //        = 2^32 - 2
                      // (32 + (2^32 - 2)) % 2^32
cout << x;            // print 30 (as expected)
```

What about the following code?

```
uint64_t x = 32;      // x can be also a pointer
x          += 2u - 4;
cout << x;
```

More negative values than positive

```
int x = std::numeric_limits<int>::max() * -1; //  (2^31 -1) * -1
cout << x;                                    //  -2^31 +1 ok

int y = std::numeric_limits<int>::min() * -1; // -2^31 * -1
cout << y; // hard to see in complex examples // 2^31 overflow!!
```

A pratical example:

```
void f(int* ptr, int pos) {
    pos++;
    if (pos < 0)
        return;      // <-- the compiler assumes that
    ptr[pos] = 0;    //     signed overflow never happen
}                    //     and removes the if statement
int main() {         // compiled with optimizations
    int tmp[10];     // leads to segmentation faults
    f(tmp, INT_MAX);
}
```

Undefined Behavior

Shift larger than #bits of the data type is undefined behavior even for **unsigned**

```
unsigned x = 1;
unsigned y = x >> 32; // undefined behavior!!
```

Undefined behavior in implicit conversion

```
uint16_t a2 = 65535; // 0xFFFF
uint16_t b2 = 65535; // 0xFFFF
cout << (a2 * b2);   // print '-131071' (0xFFFE0001)
                     // undefined behavior!! (int overflow)
```

twitter.com/shafikyaghmour/status/1134578146781491201

Even worse example:

```
#include <iostream>

int main() {
    for (int i = 0; i < 4; ++i)
        std::cout << i * 1000000000 << std::endl;
}
// with optimizations, it is an infinite loop
// --> 1000000000 * i > INT_MAX
// undefined behavior!!

// the compiler translates the multiplication constant
// into an addition
```

Why does this loop produce undefined behavior?

Is the following loop safe?

```
void f(int size) {
    for (int i = 0; i < size; i += 2)
        ...
}
```

- What happens if `size` is equal to `INT_MAX`?
- How to make the previous loop safe?
- `i >= 0 && i < size` is not the solution because of *undefined behavior* of signed overflow
- Can we generalize the solution when the increment is `i += step`?

Overflow / Underflow

Detecting overflow/underflow for unsigned integral types is **not trivial**

```
// some examples
bool is_add_overflow(unsigned a, unsigned b) {
    return (a + b) < a || (a + b) < b;
}

bool is_mul_overflow(unsigned a, unsigned b) {
    unsigned x = a * b;
    return a != 0 && (x / a) != b;
}
```

Overflow/underflow for signed integral types is **not defined** !! *Undefined behavior* must be checked before performing the operation

Floating-point Types and Arithmetic

32/64-bit Floating-Point

IEEE754 is the technical standard for floating-point arithmetic

The standard defines the binary format, operations behavior, rounding rules, exception handling, etc.

- First Release: 1985
- Second Release: 2008. Add 16-bit floating point
- Third Release: 2019. Specify `min`/`max` behavior

IEEE764 technical document:

`754-2019 - IEEE Standard for Floating-Point Arithmetic`

`The IEEE Standard 754: One for the History Books`

In general, *C/C++ adopts IEEE754 floating-point standard*:

`en.cppreference.com/w/cpp/types/numeric_limits/is_iec559`

32/64-bit Floating-Point

- **IEEE764 Single precision** (32-bit) `float`

Sign	Exponent (or base)	Mantissa (or significant)
1-bit	8-bit	23-bit

- **IEEE764 Double precision** (64-bit) `double`

Sign	Exponent (or base)	Mantissa (or significant)
1-bit	11-bit	52-bit

16-bit Floating-Point (non-standard)

- **IEEE754 16-bit Floating-point** (`fp16`)

Sign	Exponent	Mantissa
1-bit	5-bit	10-bit

- **Google 16-bit Floating-point** (`bfloat16`)

Sign	Exponent	Mantissa
1-bit	8-bit	7-bit

`half-precision-arithmetic-fp16-versus-bfloat16`

Other Real Value Representations (non-standard)

- **Posit** (John Gustafson, 2017), also called *unum III* (*universal number*), represents floating-point values with *variable-width* of exponent and mantissa

- **Fixed-point** representation has a fixed number of digits after the radix point (decimal point). The gaps between adjacent numbers are always equal. The range of their values is significantly limited compared to floating-point numbers

---

- Beating Floating Point at its Own Game: Posit Arithmetic
- Comparing posit and IEEE-754 hardware cost

Floating-point number:

- *Radix* (or base): β
- *Precision* (or digits): p
- *Exponent* (magnitude): e
- *Mantissa*: M

$$n = \underbrace{M}_{p} \times \beta^e \quad \rightarrow \quad \text{IEEE754: } 1.M \times 2^e$$

```
float  f1 = 1.3f;   // 1.3
float  f2 = 1.1e2f; // 1.1 · 10^2
float  f3 = 3.7E4f; // 3.7 · 10^4
float  f4 = .3f;    // 0.3
double d1 = 1.3;    // without "f"
double d2 = 5E3;    // 5 · 10^3
```

Exponent Bias

In IEEE754 floating point numbers, the exponent value is offset from the actual value by the **exponent bias**

- The exponent is stored as an unsigned value suitable for comparison
- Floating point values are lexicographic ordered
- For a single-precision number, the exponent is stored in the range $[1, 254]$ (0 and 255 have special meanings), and is biased by subtracting 127 to get an exponent value in the range $[-126, +127]$

0	10000111	11000000000000000000000
+	$2^{(135-127)} = 2^8$	$\frac{1}{2^1} + \frac{1}{2^2} = 0.5 + 0.25 = 0.75 \overset{normal}{\longrightarrow} 1.75$

$$+1.75 * 2^8 = 448.0$$

Normal number

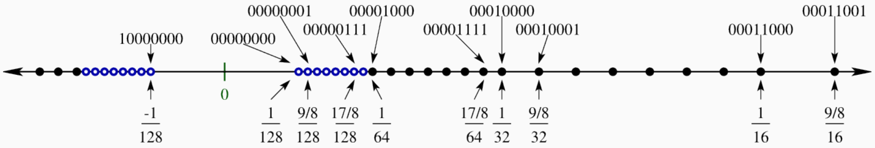
A **normal** number is a floating point value that can be represented with *at least one bit set in the exponent* or the mantissa has all 0s

Denormal number

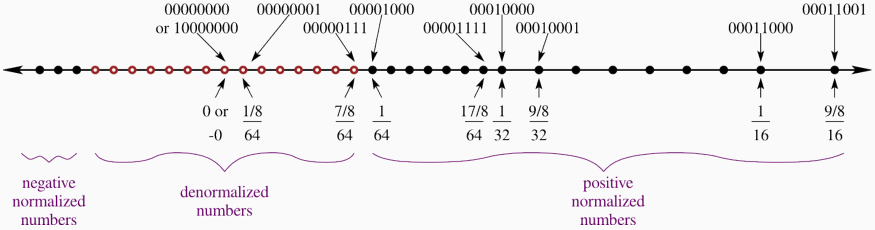
Denormal (or subnormal) numbers fill the underflow gap around zero in floating-point arithmetic. Any non-zero number with magnitude smaller than the smallest normal number is denormal

A **denormal** number is a floating point value that can be represented with *all 0s in the exponent*, but the mantissa is non-zero

Why denormal numbers make sense: (↓ normal numbers)



The problem: distance values from zero (↓ denormal numbers)



www.toves.org/books/float/

Floating-point - Special Values

- $\pm$ infinity

*	11111111	00000000000000000000000

- NaN (mantissa $\neq 0$)

*	11111111	***********************

- ± 0

*	00000000	00000000000000000000000

- Denormal number ($< 2^{-126}$)(minimum: $1.4 * 10^{-45}$)

*	00000000	***********************

- Minimum (normal) ($\pm 1.17549 * 10^{-38}$)

*	00000001	00000000000000000000000

- Lowest/Largest ($\pm 3.40282 * 10^{+38}$)

*	11111110	11111111111111111111111

Machine Epsilon

Machine epsilon

Machine epsilon ε (or *machine accuracy*) is defined to be the smallest number that can be added to 1.0 to give a number other than one

IEEE 754 Single precision : $\varepsilon = 2^{-23} \approx 1.19209 * 10^{-7}$

IEEE 754 Double precision : $\varepsilon = 2^{-52} \approx 2.22045 * 10^{-16}$

Units at the Last Place

ULP

Units at the Last Place is the gap between consecutive floating-point numbers

$$ULP(p, e) = 1.0 \times \beta^{e-(p-1)}$$

Example:

$$\beta = 10,\ p = 3$$
$$\pi = 3.1415926... \rightarrow x = 3.14 \times 10^0$$
$$ULP(3, 0) = 10^{-2} = 0.01$$

Relation with ε:

- $\varepsilon = ULP(p, 0)$
- $ULP_x = \varepsilon * \beta^{e(x)}$

Floating-point Error

Machine floating-point representation of x is denoted $\mathbf{fl}(x)$

$fl\,(x) = x\,(1 + \delta)$

Absolute Error: $|fl(x) - x| \leq \frac{1}{2} \cdot ULP_x$

Relative Error: $\left|\frac{fl\,(x) - x}{x}\right| \leq \frac{1}{2} \cdot \varepsilon$

Floating-point Summary

	half	bfloat16	float	double
exponent	5-bit [0*-30]	8-bit [0*-254]		11-bit [0*-2046]
bias	15	127		1023
mantissa	10-bit	7-bit	23-bit	52-bit
largest ($\pm$)	2^{16} $65,536$	2^{128} $3.4 \cdot 10^{38}$		2^{1024} $1.8 \cdot 10^{308}$
smallest ($\pm$)	2^{-14} 0.00006	2^{-126} $1.2 \cdot 10^{-38}$		2^{-1022} $2.2 \cdot 10^{-308}$
smallest (denormal)	2^{-24} $6.0 \cdot 10^{-8}$	/	2^{-149} $1.4 \cdot 10^{-45}$	2^{-1074} $4.9 \cdot 10^{-324}$
epsilon	2^{-10} 0.00098	2^{-7} 0.0078	2^{-23} $1.2 \cdot 10^{-7}$	2^{-52} $2.2 \cdot 10^{-16}$

Floating-point Type Limits

`T` : float or double

```
#include <limits>

// Check if the actual C++ implementation adopts
// the IEEE754 standard:
std::numeric_limits<T>::is_iec559;   // should return true

std::numeric_limits<T>::max();        // largest value

std::numeric_limits<T>::lowest();     // lowest value (C++11)

std::numeric_limits<T>::min();        // smallest value

std::numeric_limits<T>::denorm_min() // smallest (denormal) value

std::numeric_limits<T>::epsilon();   // epsilon value
```

`NaN` Properties

NaN

In the IEEE754 standard, **`NaN`** (not a number) is a numeric data type value representing an undefined or unrepresentable value

Operations generating **`NaN`**:

- Operations with a `NaN` as at least one operand
- $\pm\infty \cdot \mp\infty$, $0 \cdot \infty$
- $0/0, \infty/\infty$
- $\sqrt{x}$, $\log(x)$ for $x < 0$
- $\sin^{-1}(x), \cos^{-1}(x)$ for $x < -1$ *or* $x > 1$

There are many representations for `NaN` (e.g. $2^{24} - 2$ for `float`)

Comparison: `(NaN == x)` $\rightarrow$ `false`, for every `x`

`(NaN == NaN)` $\rightarrow$ `false`

`inf` Properties

Infinity

In the IEEE754 standard, **`inf`** (infinity value) is a numeric data type value that exceeds the maximum (or minimum) representable value

Operations generating **`inf`**:

- $\pm\infty \cdot \pm\infty$
- $\pm\infty \cdot \pm$`finite_value`
- `finite_value` *op* `finite_value` $>$ `max_value`
- `non-NaN` $/ \pm 0$

There is a single representation for `+inf` and `-inf`

Comparison: (`inf == finite_value`) $\rightarrow$ `false`

(`±inf == ±inf`) $\rightarrow$ `true`

Floating-point - Useful Functions

```
#include <limits>

std::numeric_limits<T>::infinity()  // infinity
std::numeric_limits<T>::quiet_NaN() // NaN
```

```
#include <cmath> // C++11

bool std::isnan(T value)      // check if value is NaN
bool std::isinf(T value)      // check if value is ±infinity
bool std::isfinite(T value)   // check if value is not NaN
                              // and not ±infinity

bool std::isnormal(T value); // check if value is normal

T    std::ldexp(T x, p)       // exponent shift x * 2^p
int  std::ilogb(T value)      // extracts the exponent of value
```

Floating-point Special Values Behavior

```
cout << 0 / 0;          // undefined behavior
cout << 0.0 / 0.0;      // print "nan"
cout << 5.0 / 0.0;      // print "inf"
cout << -5.0 / 0.0;     // print "-inf"

auto inf = std::numeric_limits<float>::infinity;
cout << (-0.0 == 0.0);                     // true
cout << ((5.0f / inf) == ((-5.0f / inf));  // true
cout << (10e40f) == (10e40f + 9999999.0f); // true
cout << (10e40)  == (10e40f + 9999999.0f); // false
```

Floating-point Arithmetic Properties 1/2

Floating-point operations are written

- $\oplus$ addition
- $\ominus$ subtraction
- $\otimes$ multiplication
- $\oslash$ division

$\odot \in \{\oplus, \ominus, \otimes, \oslash\}$

$op \in \{+, -, *, \backslash\}$ denotes exact precision operations

Floating-point Arithmetic Properties

(P1) In general, $a \ op \ b \neq a \odot b$

(P2) **Not Reflexive** $a \neq a$

- *Reflexive* without `NaN`

(P3) **Not Commutative** $a \odot b \neq b \odot a$

- *Commutative* without `NaN` (`NaN` $\neq$ `NaN`)

(P4) In general, **Not Associative** $(a \odot b) \odot c \neq a \odot (b \odot c)$

(P5) In general, **Not Distributive** $(a \oplus b) \otimes c \neq (a \cdot c) \oplus (b \cdot c)$

(P6) **Identity on operations is not ensured** $(k \oslash a) \otimes a \neq a$

(P7) **No overflow/underflow** Floating-point has *"saturation"* values `inf`, `-inf`

- Adding (or subtracting) can "saturate" before `inf`, `-inf`

Detect Floating-point Errors ★

C++11 allows determining if a floating-point exceptional condition has occurred by using floating-point exception facilities provided in `<cfenv>`

```
#include <cfenv>
// MACRO
FE_DIVBYZERO  // division by zero
FE_INEXACT    // rounding error
FE_INVALID    // invalid operation, i.e. NaN
FE_OVERFLOW   // overflow (reach saturation value +inf)
FE_UNDERFLOW  // underflow  (reach saturation value -inf)
FE_ALL_EXCEPT // all exceptions

// functions
std::feclearexcept(FE_ALL_EXCEPT); // clear exception status
std::fetestexcept(<macro>);        // returns a value != 0 if an
                                   // exception has been detected
```

Detect Floating-point Errors ★

```
#include <cfenv>     // floating point exceptions
#include <iostream>
#pragma STDC FENV_ACCESS ON // tell the compiler to manipulate the floating-point
                            // environment (not supported by all compilers)
                            // gcc: yes, clang: no
int main() {
    std::feclearexcept(FE_ALL_EXCEPT); // clear
    auto x = 1.0 / 0.0;                // all compilers
    std::cout << (bool) std::fetestexcept(FE_DIVBYZERO); // print true

    std::feclearexcept(FE_ALL_EXCEPT); // clear
    auto x2 = 0.0 / 0.0;               // all compilers
    std::cout << (bool)  std::fetestexcept(FE_INVALID);  // print true

    std::feclearexcept(FE_ALL_EXCEPT); // clear
    auto x4 = 1e38f * 10;              // gcc: ok
    std::cout << std::fetestexcept(FE_OVERFLOW);         // print true
}
```

see What is the difference between quiet NaN and signaling NaN?

Floating-point Issues

Ariene 5: data conversion from 64-bit floating point value to 16-bit signed integer → *$137 million*

Patriot Missile: small chopping error at each operation, 100 hours activity → *28 deaths*

Integer type is more accurate than floating type for large numbers

```
cout << 16777217;            // print 16777217
cout << (int) 16777217.0f; // print 16777216!!
cout << (int) 16777217.0;  // print 16777217, double ok
```

`float` numbers are different from `double` numbers

```
cout << (1.1 != 1.1f); // print true !!!
```

The floating point precision is finite!

```
cout << setprecision(20);
cout << 3.33333333f; // print 3.333333254!!
cout << 3.33333333;  // print 3.333333333
cout << (0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1);
// print 0.59999999999999998
```

Floating point arithmetic is not associative

```
cout << 0.1 + (0.2 + 0.3) == (0.1 + 0.2) + 0.3; // print false
```

IEEE764 Floating-point computation guarantees to produce **deterministic** output, namely the exact bitwise value for each run, if and only if the **order of the operations is always the same**

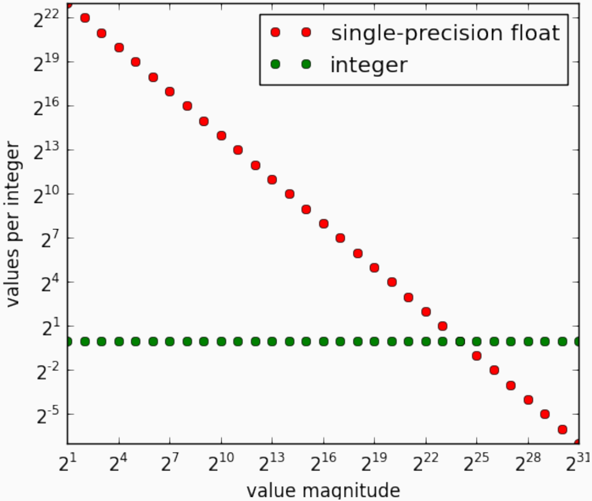
→ *same result on any machine and for all runs*

"Using a double-precision floating-point value, we can represent easily the number of atoms in the universe.

If your software ever produces a number so large that it will not fit in a double-precision floating-point value, chances are good that you have a bug"

***Daniel Lemire**, Prof. at the University of Quebec*

`Number of atoms in the universe versus floating-point values`



Intersection $= 16,777,216 = 2^{24}$

Floating-point increment

```
float x = 0.0f;
for (int i = 0; i < 20000000; i++)
    x += 1.0f;
```

What is the value of `x` at the end of the loop?

Ceiling division $\left\lceil \frac{a}{b} \right\rceil$

```
//        std::ceil((float) 101 / 2.0f) -> 50.5f -> 51
float x = std::ceil((float) 20000001 / 2.0f);
```

What is the value of `x`?

The problem

```
cout << (0.11f + 0.11f < 0.22f); // print true!!
cout << (0.1f + 0.1f > 0.2f);    // print true!!
```

Do not use absolute error margins!!

```
bool areFloatNearlyEqual(float a, float b) {
    if (std::abs(a - b) < epsilon); // epsilon is fixed by the user
        return true
    return false;
}
```

Problems:

- Fixed `epsilon` "looks small" but it could be too large when the numbers being compared are very small
- If the compared numbers are very large, the `epsilon` could end up being smaller than the smallest rounding error, so that the comparison always returns `false`

Solution: Use relative error $\frac{|a-b|}{b} < \varepsilon$

```
bool areFloatNearlyEqual(float a, float b) {
    if (std::abs(a - b) / b < epsilon); // epsilon is fixed
        return true
    return false;
}
```

Problems:

- `a=0, b=0` The division is evaluated as `0.0/0.0` and the whole `if` statement is `(nan < espilon)` which always returns `false`
- `b=0` The division is evaluated as `abs(a)/0.0` and the whole `if` statement is `(+inf < espilon)` which always returns `false`
- **`a and b very small.`** The result should be `true` but the division by `b` may produces wrong results
- **`It is not commutative.`** We always divide by `b`

Possible solution: $\frac{|a-b|}{\max(|a|,|b|)} < \varepsilon$

```cpp
bool areFloatNearlyEqual(float a, float b) {
    const float normal_min     = std::numeric_limits<float>::min();
    const float relative_error = <user_defined>

    if (std::isfinite(a) || isfinite(b)) // a = ±∞, b = ±∞ and NaN
        return false;
    float diff  = std::abs(a - b);
    // if "a" and "b" are near to zero, the relative error is less effective
    if (diff <= normal_min)
        return true; // or also: user_epsilon * normal_min

    float abs_a = std::abs(a);
    float abs_b = std::abs(b);
    return (diff / std::max(abs_a, abs_b)) <= relative_error;
}
```

Floating-point Algorithms

- `addition` **algorithm** (simplified):

(1) Compare the exponents of the two numbers. Shift the smaller number to the right until its exponent would match the larger exponent

(2) Add the mantissa

(3) Normalize the sum if needed (shift right/left the exponent)

- `multiplication` **algorithm** (simplified):

(1) Multiplication of mantissas. The number of bits of the result is twice the size of the operands (46 + 2 bits, +2 for implicit normalization)

(2) Normalize the product if needed (shift right/left the exponent)

(3) Addition of the exponents

- `fused multiply-add` (`fma`):
  - Recent architectures (also GPUs) provide `fma` to compute these two operations in a single instruction (performed by the compiler)
  - The rounding error is lower $fl(fma(x, y, z)) < fl((x \otimes y) \oplus z)$

Catastrophic Cancellation

Catastrophic cancellation (or *loss of significance*) refers to loss of relevant information in a floating-point computation that cannot be revered

Two cases:

(1) $\mathbf{a} \pm \mathbf{b}$, where $\mathbf{a} \gg \mathbf{b}$ or $\mathbf{b} \gg \mathbf{a}$. The value (or part of the value) of the smaller number is lost

(2) $\mathbf{a} - \mathbf{b}$, where $\mathbf{a} \approx \mathbf{b}$. Loss of precision in both a and b. It implies large relative error

Catastrophic Cancellation (case 1) 2/3

How many iterations performs the following code?

```
while (x > 0)
    x = x - y;
```

```
float:  x = 10,000,000   y = 1     -> 10,000,000
float:  x = 30,000,000   y = 1     -> does not terminate
float:  x =    200,000   y = 0.001 -> does not terminate
bfloat: x =        256   y = 1     -> does not terminate !!
```

Let's solve a quadratic equation:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x^2 + 5000x + 0.25 \qquad x_{1,2} = 0.00005, -5000$

```
(-5000 + std::sqrt(5000.0f * 5000.0f - 4.0f * 1.0f * 0.25f)) / 2
(-5000 + std::sqrt(25000000.0f - 1.0f)) / 2 // !! case 1
(-5000 + std::sqrt(25000000.0f)) / 2
(-5000 + 5000) / 2 = 0                       // !! case 2
```

relative error: $\frac{|0 - 0.00005|}{0.00005} = 100\%$

Minimize Error Propagation

- Prefer **multiplication/division** rather than addition/subtraction
- Scale by a **power of two** is safe
- Try to reorganize the computation to **keep near** numbers with the same scale (e.g. sorting numbers)
- Consider to **put a zero** very small number (under a threshold). Common application: iterative algorithms
- **Switch to log scale**. Multiplication becomes Add, and Division becomes Subtraction

References

Suggest readings:

- `What Every Computer Scientist Should Know About Floating-Point Arithmetic`
- `Do Developers Understand IEEE Floating Point?`
- `Yet another floating point tutorial`
- `Unavoidable Errors in Computing`

Floating-point Comparison readings:

- `The Floating-Point Guide - Comparison`
- `Comparing Floating Point Numbers, 2012 Edition`
- `Some comments on approximately equal FP comparisons`
- `Comparing Floating-Point Numbers Is Tricky`

Floating point online visualization tool:

`www.h-schmidt.net/FloatConverter/IEEE754.html`

On Floating-point